

## Reply to "Comment on 'Oscillation mode and "nonlinear" radiation of the double sine-Gordon $2\pi$ kink'"

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The  $2\pi$  kink is by no means stable. The linear stability analysis of the collective coordinate  $\Delta$  according to its definition characterizing the relative motion of two  $\pi$  kinks was intended to find conditions of linear stability of each of two  $\pi$  kinks when perturbed by the coupling with the other. The relativistic invariance of  $\Delta$  and  $\Omega$  is evident from the formulas (6) and (12) of the paper [E. Majerníková, Phys. Rev. E **49**, 3360 (1994)]. In Sec. III of that paper there are clearly stated conditions of the approximate analytic calculations, including also  $\dot{x}_0^2 \ll 1$ .

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(i) In the analysis of the double sine-Gordon equation

$$\phi_{tt} - \phi_{xx} + \lambda \sin\phi + \sin 2\phi = 0 \quad (1)$$

presented in the paper [1], I used a parametrized ansatz for the solution (solitary wave)

$$\phi = 2 \arctan[\exp(\theta + \Delta)] + 2 \arctan[\exp(\theta - \Delta)], \quad (2)$$

where  $\cosh^2 \Delta = 1 + 2/\lambda$  was obtained by substitution of (2) into (1). Therefore there cannot be any doubt that the solution (2) is exact. In the paper [1] it states "...it is reasonable to choose the solution to (1) in the form..." This means that in the analysis that followed I preferred the form (2) rather than the equivalent forms given by Eq. (4) of the paper [1]. However, full solution must include also oscillation modes. It is known that with the exception of  $\lambda = 0$ , Eq. (1) is not integrable and does not have soliton solutions.

(ii) In the paper [1] a collective coordinate  $\Delta$  which represents relative motion of two  $\pi$  solitons has been introduced as a respective degree of freedom. The oscillations of  $\Delta$  describe internal oscillations of the  $2\pi$  kink. It has been shown that the oscillation mode is linearly unstable for  $\lambda > 0.774$ . For  $\lambda < 0.774$  two  $\pi$  solitons oscillate around their center while for  $\lambda > 0.774$  the mode

becomes unstable and then after the collision the kinks radiate energy [2–4]. In this sense internal motion of the two coupled  $\pi$  kinks is unstable.

(iii) The relativistic invariance of  $\Delta$  is evident from the formula (6) of the paper [1], where the argument of (2) has been generalized as

$$\theta \pm \Delta \rightarrow (2 + \lambda)^{1/2} [x - x_0(t) \pm \Delta(t)] / (1 - \dot{x}_0^2)^{1/2}.$$

As was noted at the beginning of Sec. III of Ref. [1], the result for the frequency  $\Omega$  {Eq. (20) in Ref. [1]} has been looked for in a simplified case when  $\ddot{x}_0 = 0$  and  $\dot{x}_0^2 \ll 1$ , so that the relativistic invariance, present in the exact Hamiltonian (12),

$$\begin{aligned} H = E_0 + \frac{m(\Delta_0)\dot{x}_0^2}{2(1-\dot{x}_0^2)} & \left( 1 + \left[ 1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2} \right]^2 \right) \\ & + \frac{M(\Delta_0)}{2(1-\dot{x}_0^2)} \left[ 1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2} \right]^2 i^2 + \frac{2(2+\lambda)^{1/2}}{(1-\dot{x}_0^2)} \\ & \times \sum_{n=1}^{\infty} \frac{1}{n!} \left( f_{1n}\dot{x}_0^2 \left[ 1 + \frac{x_0\ddot{x}_0}{1-\dot{x}_0^2} \right]^2 + g_n \right) l^n, \end{aligned}$$

was lost due to neglecting the respective terms.

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